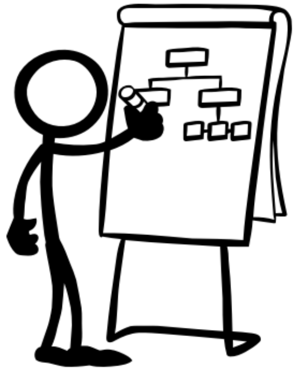
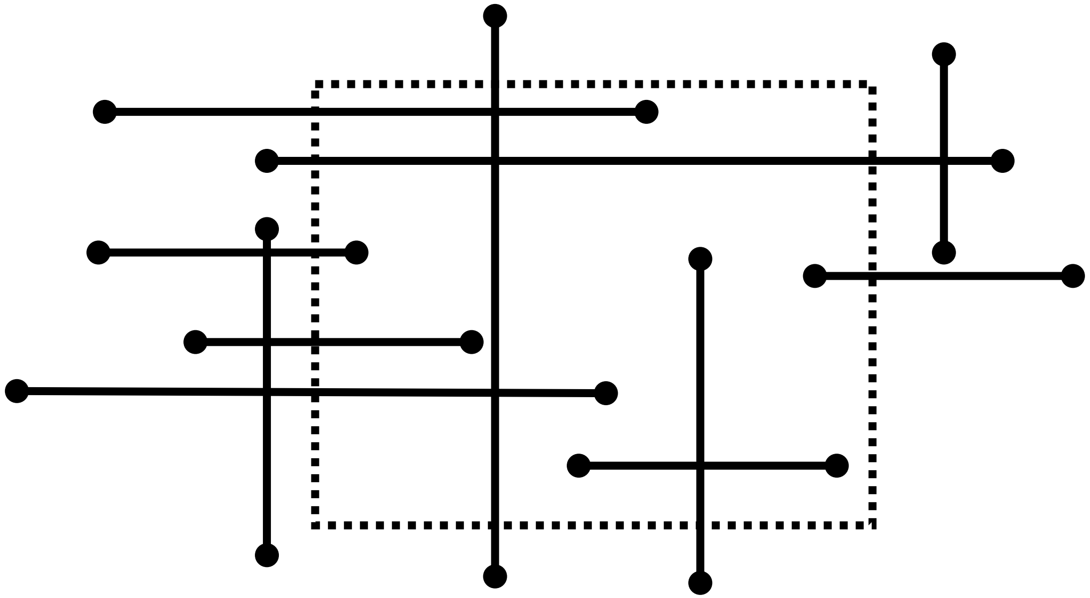
Some More Geometric Data Structures (Windowing cont.)

Computational Geometry – Recitation 12



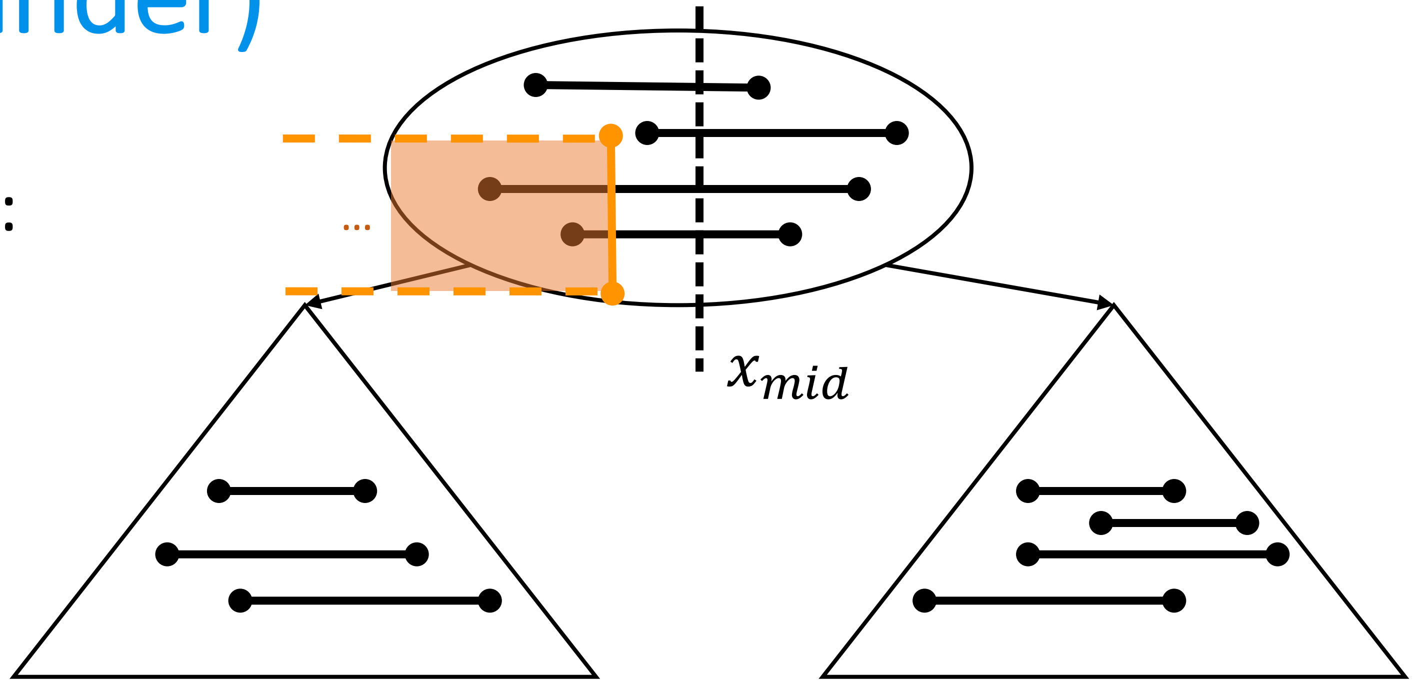
Windowing (reminder)

- We have seen how to find axis-aligned lines intersecting an axis-aligned window.



Interval trees (reminder)

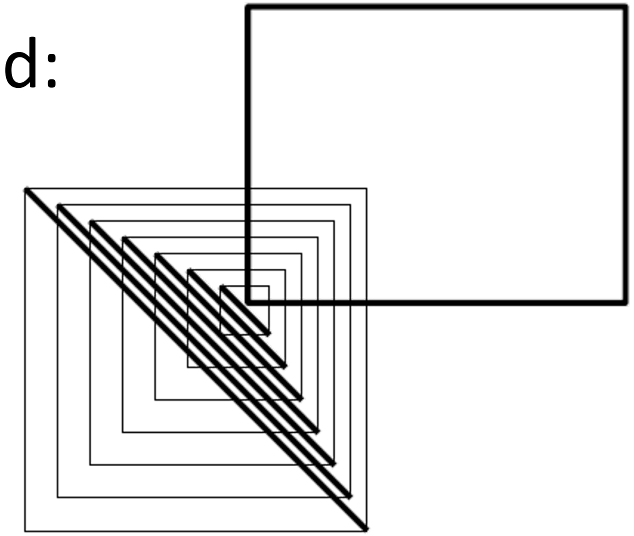
- We have used interval trees:

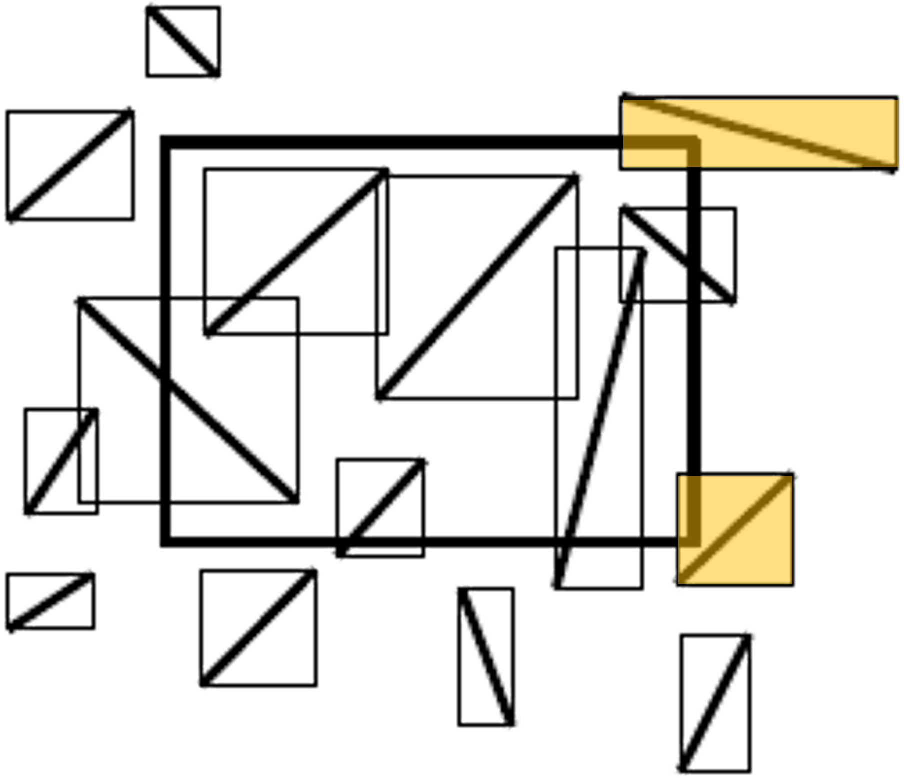


- In the relevant nodes we searched for the end points contained in a rectangle unbounded from one side.
- For this we have used 2d-Range Trees and then improved to Priority Search Trees.

Non-Axis-Aligned segments

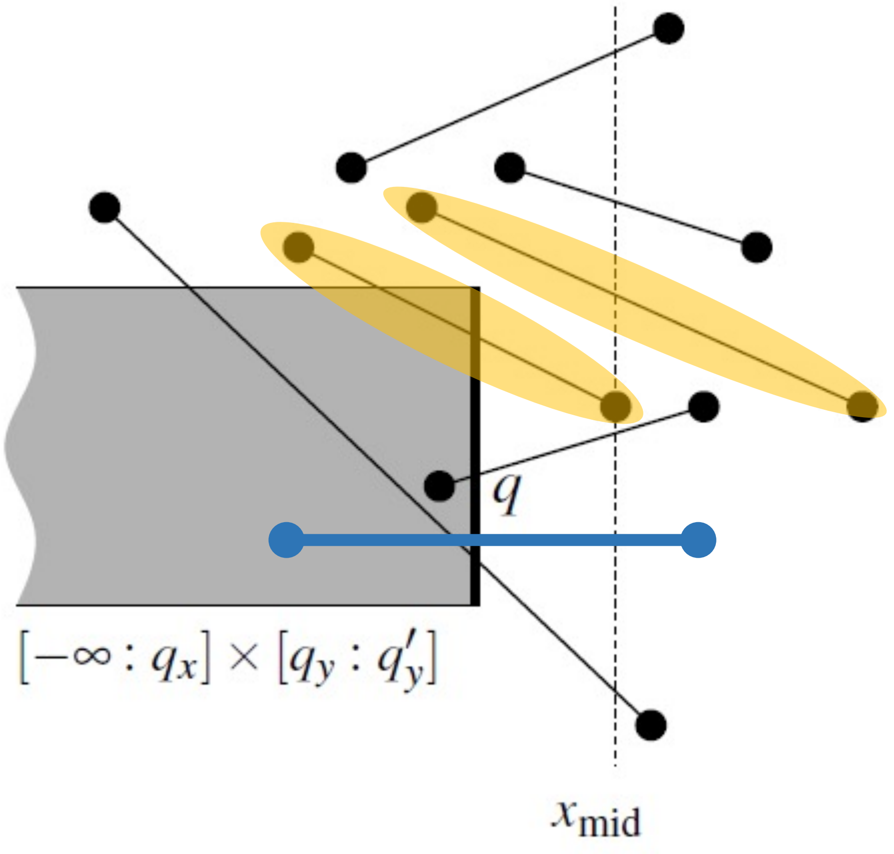
- What about general segments, that is, not axis-aligned?
  - We will restrict the problem to non-intersecting segments.
- Can we use the solution we already have?
- Use segment bounding box instead!
- Works quite well in practice.
- Worst case is bad:





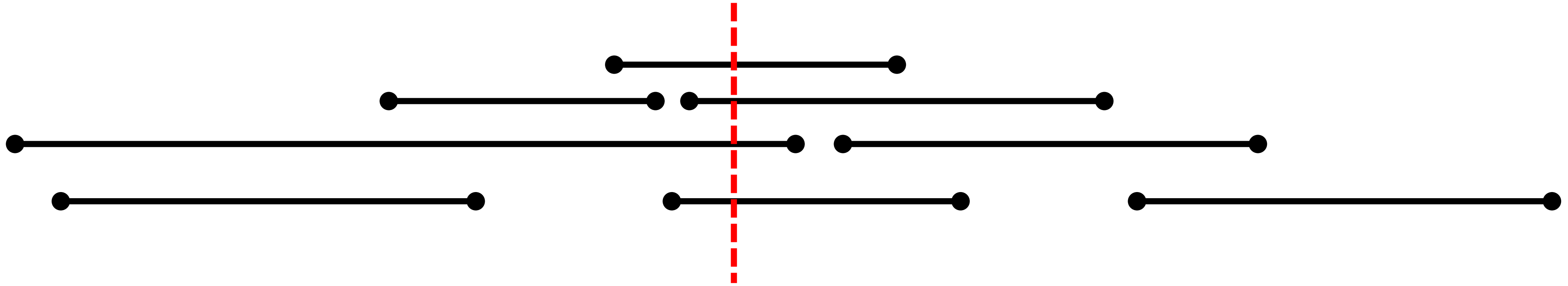
Non Axis-Aligned segments

- Can we adopt interval trees?
- The key point in interval trees is knowing that one side of the segment is to the right (or left) of q.
- This doesn't help much if we allow arbitrary orientation.



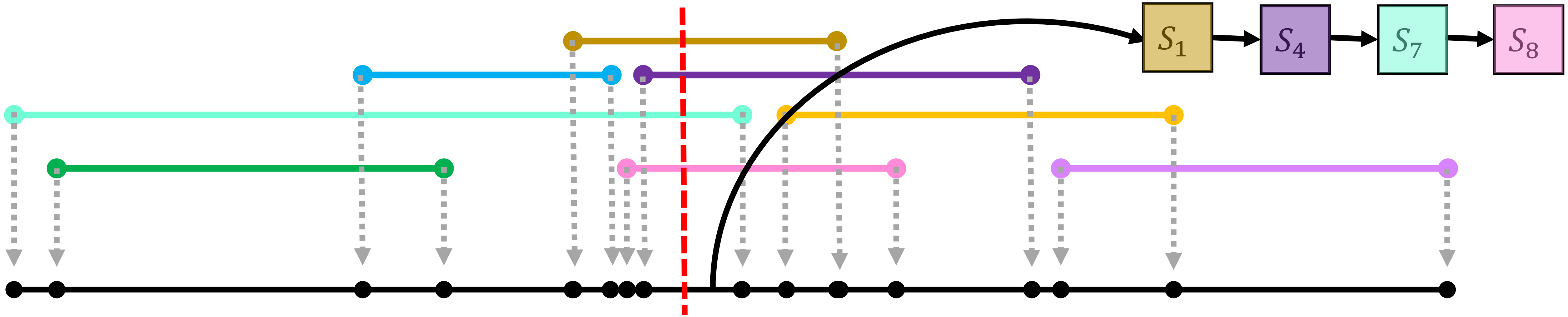
Segment trees

- Let’s remember what interval trees solves in the first place:
- Finding the $1d$-segments that cover a given point x.
- Can we devise another data structure for that?
- If the segments doesn’t overlap we can store them in a BST, and looking for the one segment that intersects x is easy.
- But what if they do overlap?



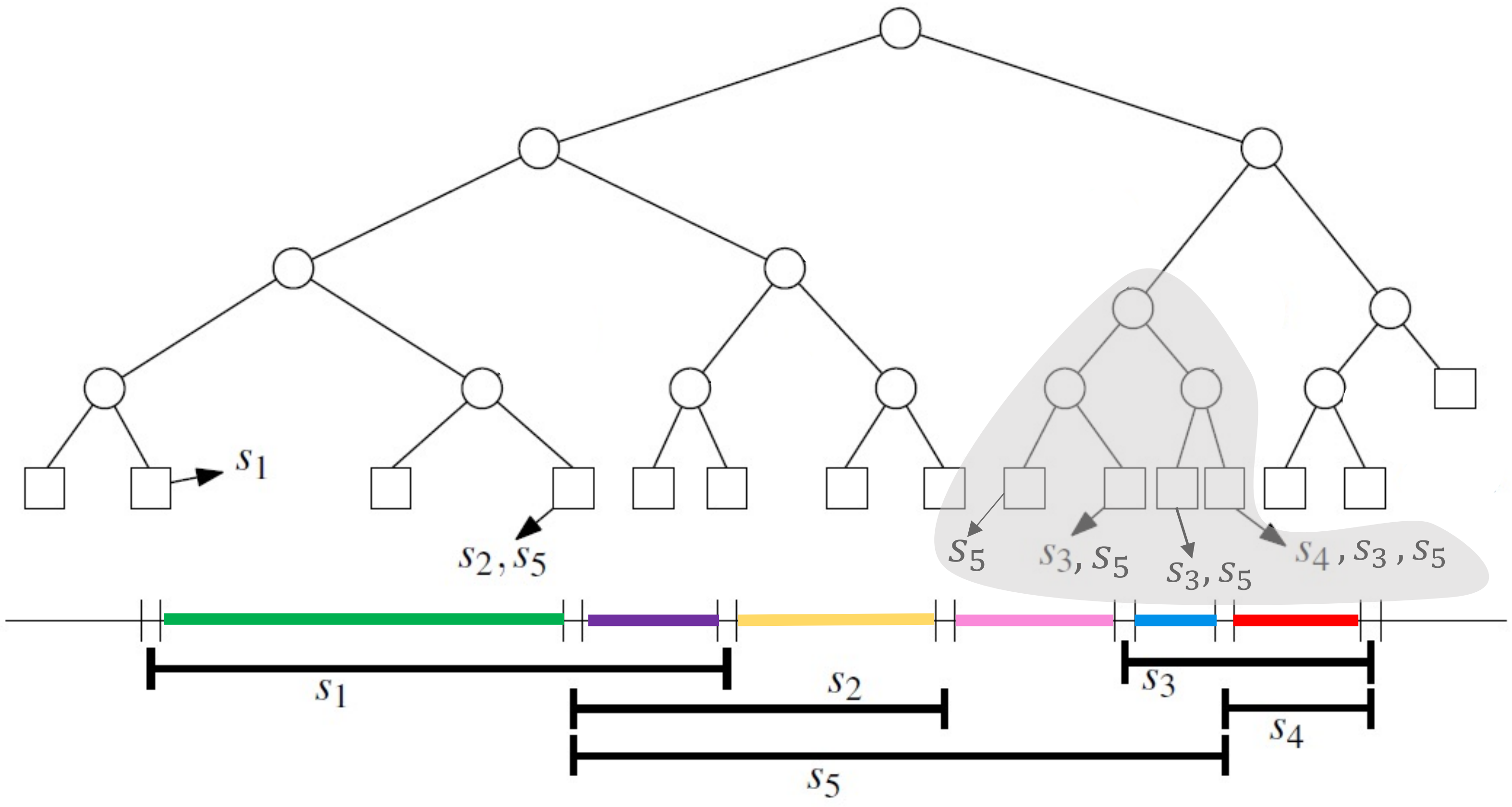
Segment trees

- Given a set S of overlapping segments, we want to find which segments intersects a point x.
- Create a new set, of non overlapping segments and store it in a BST.
  - Add zero-size segments for the end points.
- In each leaf store a list of (original) segments that intersects it.

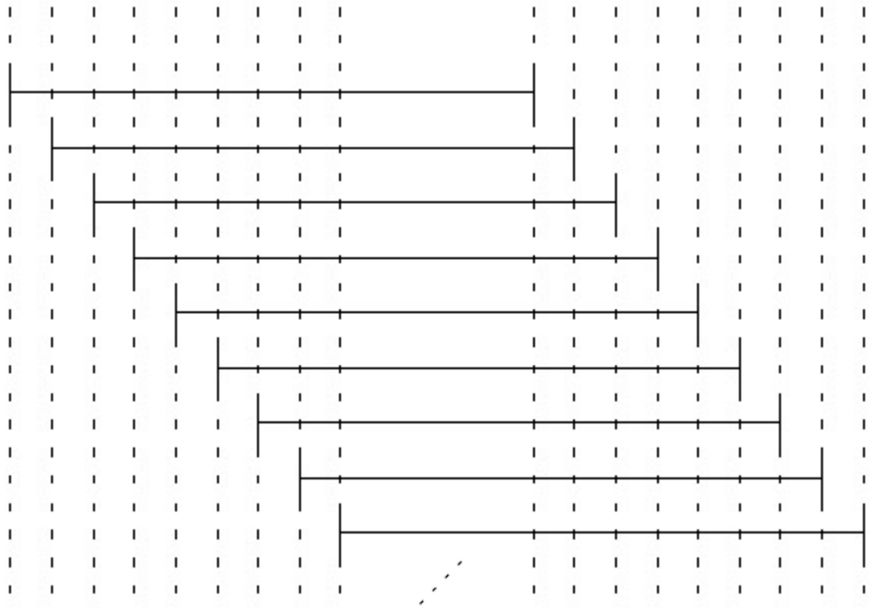


Segment trees

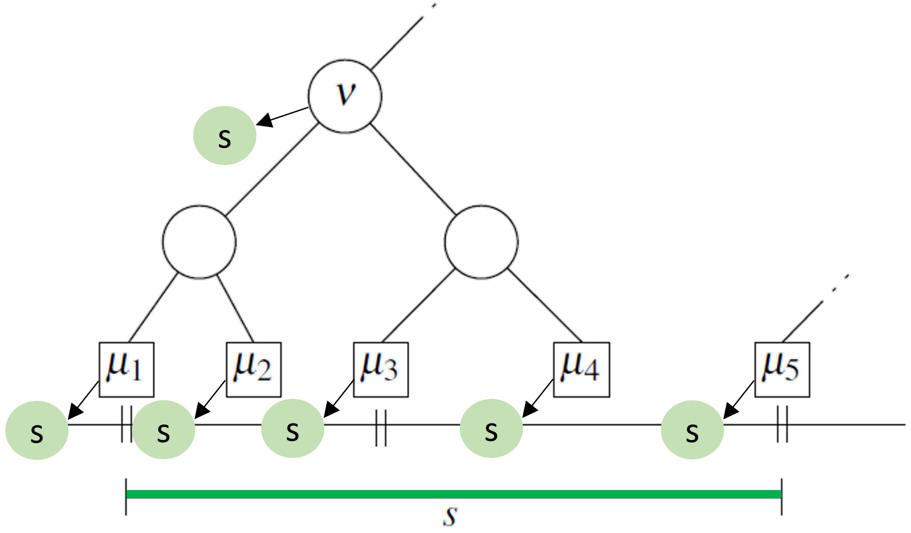
- We will save these segments in a searching tree



Segment trees – (space complexity)

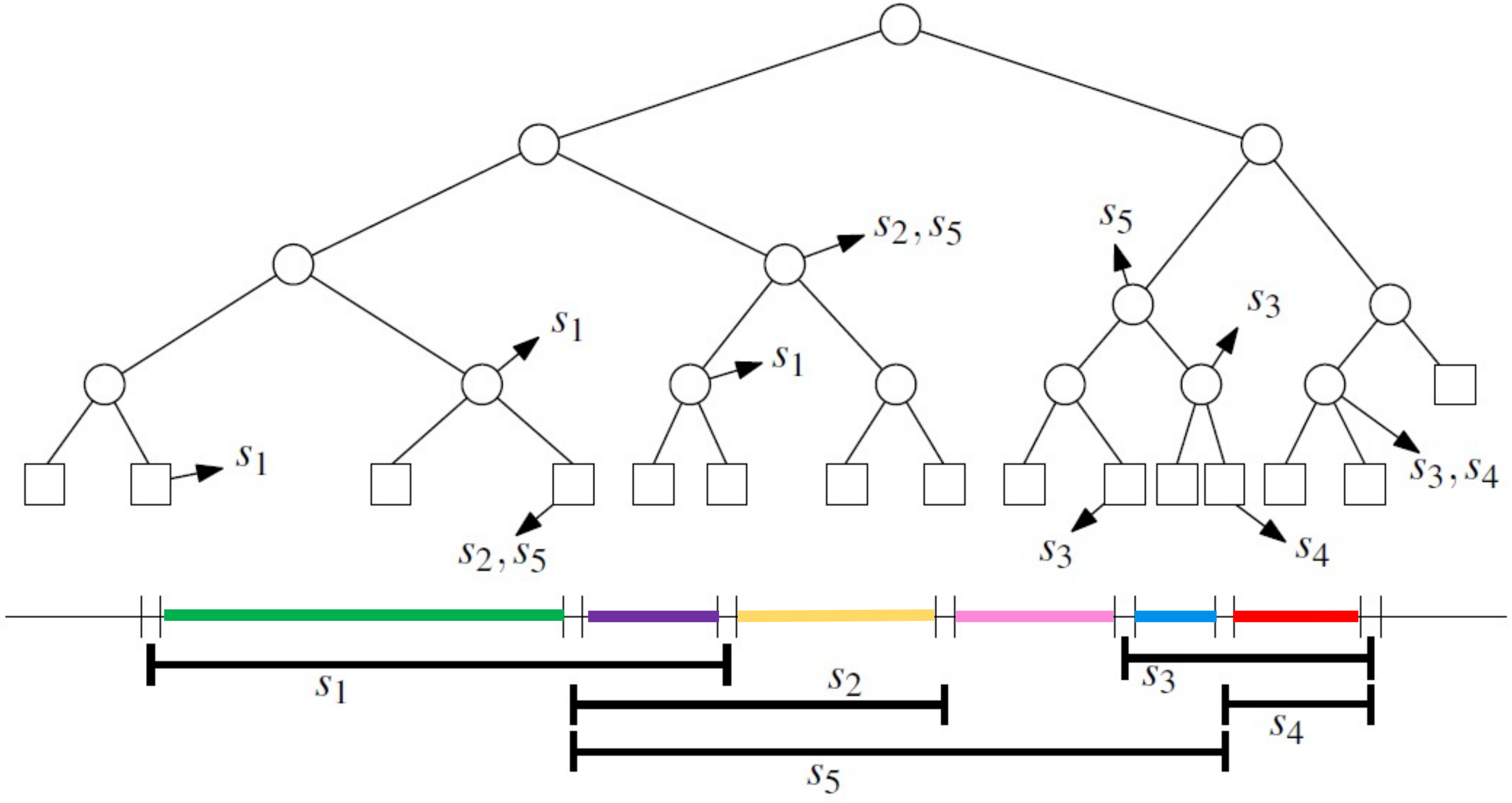


- What is the space complexity of this data structure?
- Each segment can appear in many leaves.
- The space complexity is $O(n^2)$.
- Can we improve it?
- If a segment appear in consecutive leaves, we can store it in the parent node instead.
- s will be stored in v and μ_5.



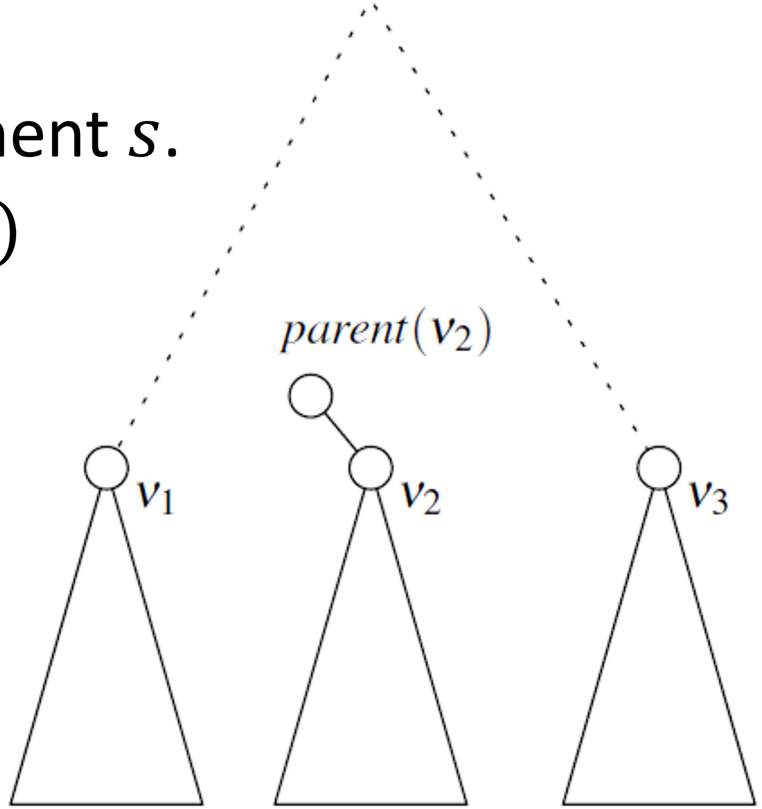
Segment trees

- The complete data structure:



Segment trees (space complexity)

- What is the space complexity now?
- Each segment can appear at most twice at any level of the tree.
- **Proof:** Assume to the contrary:
- All the leaves between v_1 and v_3 contain a segment s.
- Then, all the leaves in the subtree of $parent(v_2)$ also contain s, thus s will appear in $parent(v_2)$ and not in v_2.
- Conclusion: each segment is stored in $O(\log n)$ nodes.
- The space complexity is $O(n \log n)$.



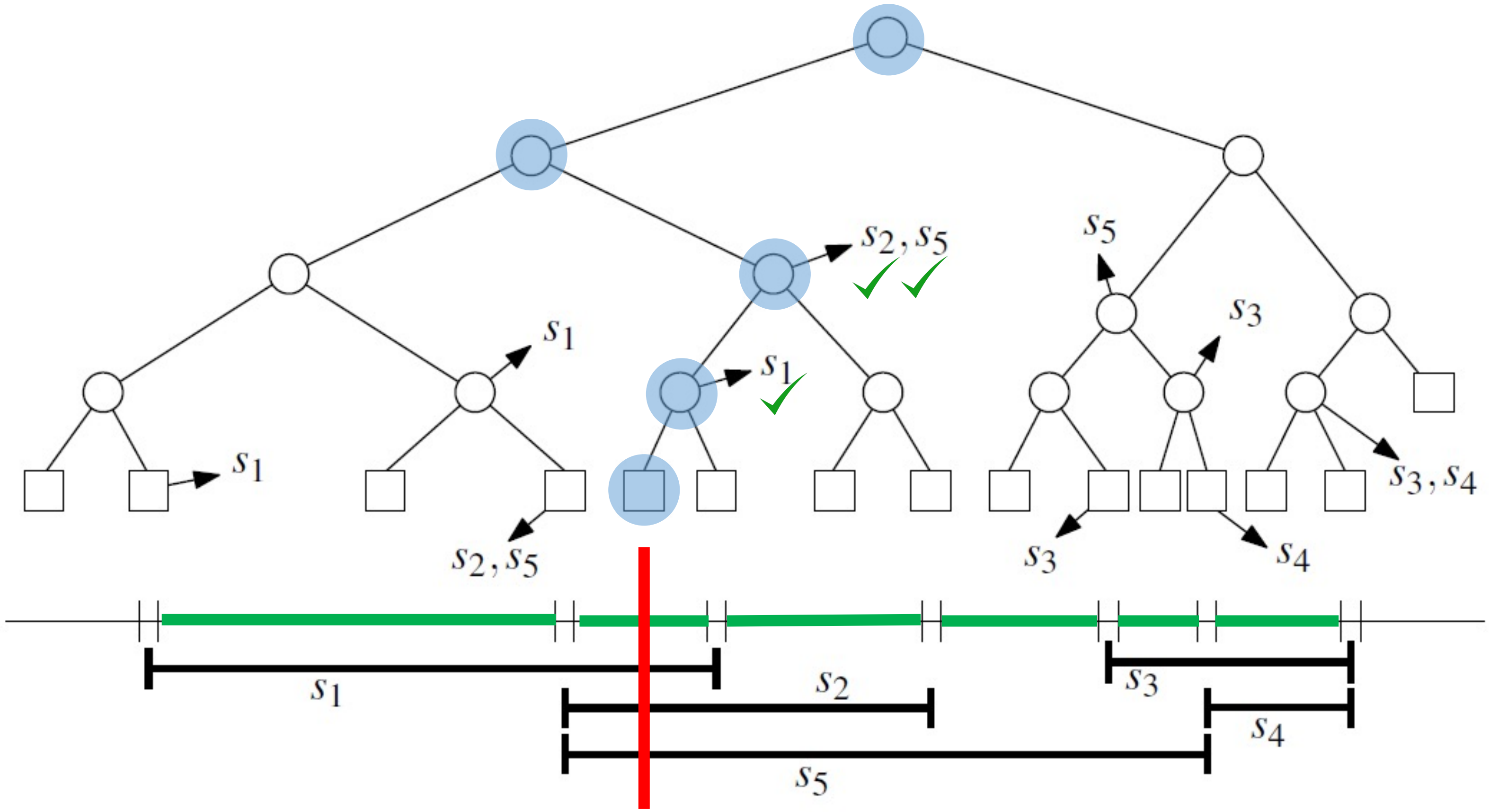
Segment trees

- Building a segment tree can also be done in $O(n \log n)$.
- How do we find all the segments covering x?
- Search for x in the tree, report all the segment stored in nodes along the search path. (next slide)
- Notice that a segment tree does the same job as a plain interval tree, but with worse space complexity.

Segment trees - Query

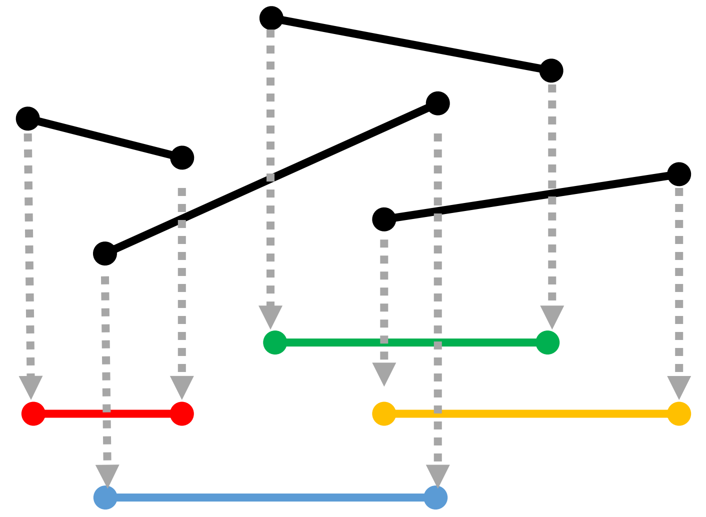
Complexity: $O(\log n + k)$
- k is the number of reported segments.

- The complete data structure:



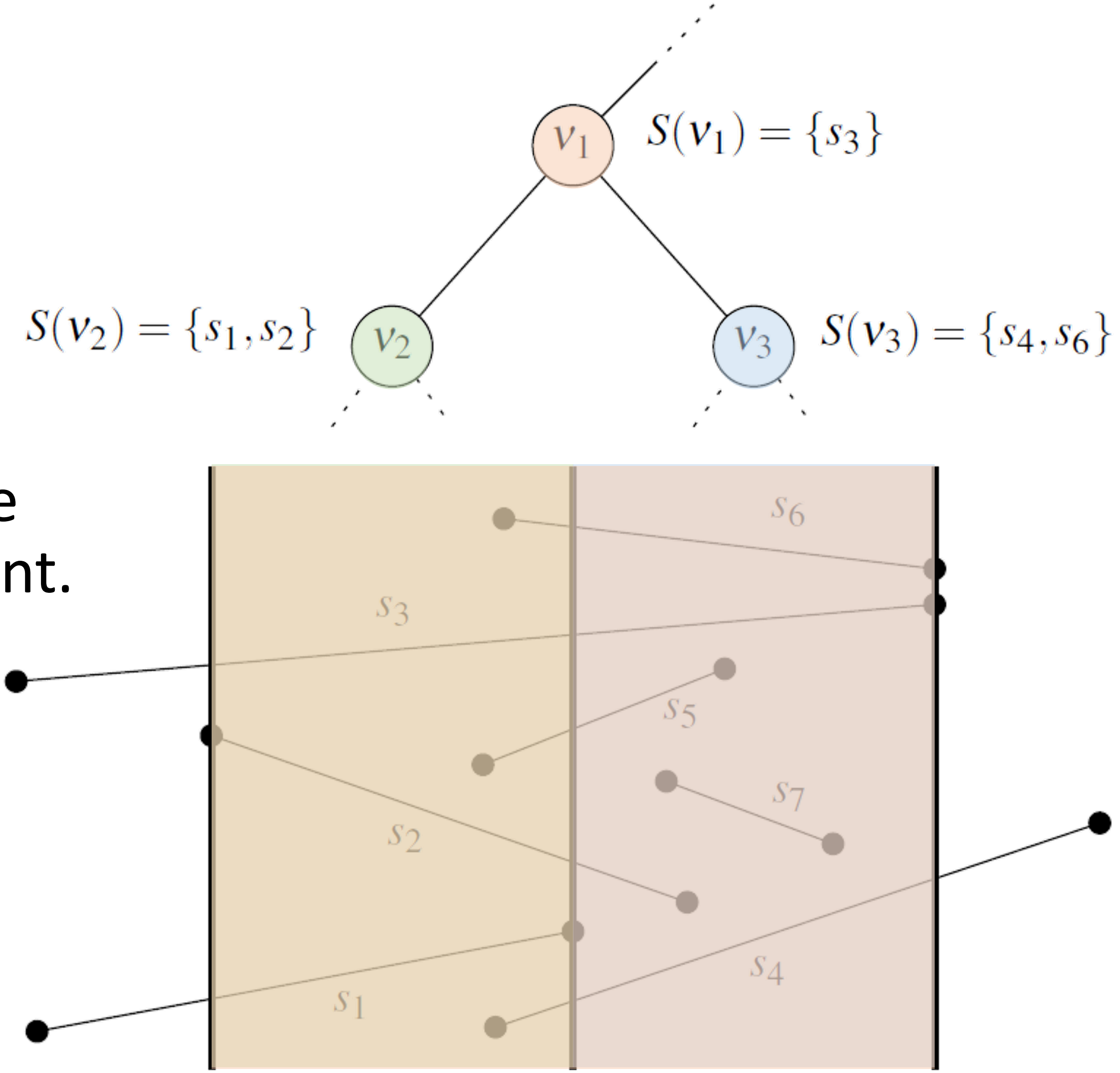
Segment trees

- So how does segment trees help us?
- Given a set of non-intersecting segments, build a segment tree to their projection on the x-axis.
- Using that we can find *potential segments*.
  Segments that cover the x coordinate of the window edge.
- How does this help?



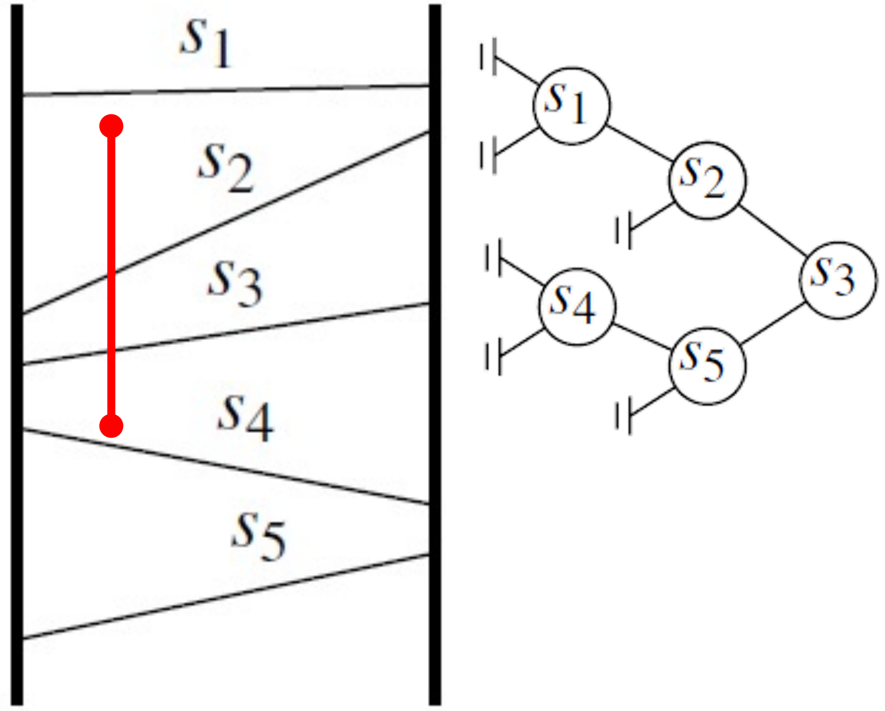
Segment trees

- Each internal node represents the union of segments of its sons.
- A segment will be stored in a node if it covers the whole node-segment.
- This means that the set of segments stored in the node is well ordered.



Segment trees

- The set of segments in each node is well ordered.
  - Intuition: it looks like a (bended) ladder.
- How can we this to find which segments intersect the window edge?
- Store the segments in a BST!



Segment trees - Complexity

- The space complexity is not affected: $O(n \log n)$
- The search in each node is done in $O(\log n)$, thus, the query complexity is $O(\log^2 n + k)$
- Building the tree takes $O(n \log^2 n)$.
- It can be improved to $O(n \log n)$ using some tricks.

Good Luck!